University of Colorado, Colorado Springs
Center for Cognitive Archaeology
ANTH 4515 / 5515: The Archaeology of Numbers

Professor: Karenleigh A. Overmann, DPhil, Oxon
Email: koverman@uccs.edu
Phone: 719-332-7183
Office Hours: Coordinate appointments via email.

OVERVIEW

Numbers are integral to daily life: We use them while shopping, telling time, placing phone calls, and driving cars. This course will introduce you to the ABCs of numbers—their archaeological, biological, cultural, and linguistic aspects. We will look at how such cultural systems are invented, how and why numbers vary across systems and change with systems over time, how societies use numbers, how having numbers changes societies, and much more.

This is *not* a math course! However, basic arithmetical competency (adding, subtracting, multiplying, and dividing small numbers) is recommended (and hoped-for). Calculators are permitted and encouraged, though most of you will find them unnecessary for the simple calculations involved in understanding what numbers are and how they vary and change.

TOPICS AND ASSIGNMENTS

Week 1	**Numbers in a nutshell:** What's a "natural" number? Cardinality (how many of something there are) vs. ordinality (how numbers are ordered). What's a number: zero and "non-being"; one and "the unity." How high can you count? "Highest number counted" vs infinity. Highest number counted and numerical elaboration. Numerical change over time and historical cumulatively. An analytical framework for comparing number systems across cultures. • **Reading** − Bender, A., & Beller, S. (2011). Cultural variation in numeration systems and their mapping onto the mental number line. *Journal of Cross-Cultural Psychology*, *42*(4), 579–597. − Chrisomalis, S. (2016). Umpteen reflections on indefinite hyperbolic numerals. *American Speech*, *91*(1), 3–33. − Núñez, R. E. (2011). No innate number line in the human brain. *Journal of Cross-Cultural Psychology*, *42*(4), 651–668. Assignment: Online comprehension quiz
Week 2	**Converging perspectives on numbers:** Culture, language, psychology, and archaeology. Historical ideas about where numbers come from: realism, intuitionism, and language. Historical change in the study of number systems. The mystery of numerical origins and the contributions and challenges of archaeology. • **Reading** − Malafouris, L. (2010). Grasping the concept of number: How did the sapient mind move beyond approximation? In C. Renfrew & I. Morley (Eds.), *The archaeology of measurement: Comprehending heaven, earth and time in ancient societies* (pp. 35–42). Cambridge: Cambridge University Press. − Núñez, R. E. (2009). Numbers and arithmetic: Neither hardwired nor out there. *Biological Theory*, *4*(1), 68–83. − Read, D. (2016). The counting numbers are a cultural idea system: A comment on Overmann 2015. *Current Anthropology*, *57*(3), 349–350. • Assignment: Online comprehension quiz
Week 3	**The role of the brain in numbers:** The perceptual experience of quantity: numerosity, subitization, and magnitude appreciation. The "mental number line." The role of processes like categorization and abstraction and how they factor into numerical origins, use, and similarities.

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	• **Reading** – Christie, S., & Gentner, D. (2007). Relational similarity in identity relation: The role of language. In S. Vosniadou, D. Kayser, & A. Protopapas (Eds.), *Proceedings of the 2nd European Cognitive Science conference* (pp. 402–406). Mahwah, NJ: Lawrence Erlbaum Associates. – Dehaene, S., Piazza, M., Pinel, P., & Cohen, L. (2003). Three parietal circuits for number processing. *Cognitive Neuropsychology*, *20*(3), 487–506. – Kaufman, E. L., Lord, M. W., Reese, T. W., & Volkmann, J. (1949). The discrimination of visual number. *American Journal of Psychology*, *62*(4), 498–525. • Assignment: Online comprehension quiz
Week 4	**Behaviors and bodies in numbers:** Why everyone counts on their fingers. The role of culture in finger-counting. Motor movements and "mental abacus." Pairing and one-to-one correspondence. Building numbers: numerical bases and the prevalence of decimal, quinary, and vigesimal. • **Reading** – Andres, M., Di Luca, S., & Pesenti, M. (2008). Finger counting: The missing tool? *Behavioral and Brain Sciences*, *31*(6), 642–643. – Comrie, B. (2013). Numeral bases. In M. S. Dryer & M. Haspelmath (Eds.), *World atlas of language structures online*. Retrieved from http://wals.info/chapter/131 – Frank, M. C., & Barner, D. (2012). Representing exact number visually using mental abacus. *Journal of Experimental Psychology: General*, *141*(1), 134–149. – Sizer, W. S. (1991). Mathematical notions in preliterate societies. *The Mathematical Intelligencer*, 13(4), 53–60. • Assignment: Online comprehension quiz
Week 5	**Materiality in numbers:** Not just fingers or notations but other forms of materiality for counting and calculating: toes and other body parts, notched tallies, torn leaves, knotted strings, beads, pebbles, grain, body stripes, marks on the ground. Role of materiality in concepts. Material properties and numerical structure. Compounding strategies; grouping, over- and back-counting. Spoken, written, and unwritten forms. • **Reading** – Chrisomalis, S. (2004). A cognitive typology for numerical notation. *Cambridge Archaeological Journal*, *14*(1), 37–52. – Greeno, J. G. (1994). Gibson's affordances. *Psychological Review*, *101*(2), 336–342. – Hutchins, E. (2005). Material anchors for conceptual blends. *Journal of Pragmatics*, *37*(10), 1555–1577. • Assignment: Online comprehension quiz
Week 6	**Language in numbers:** Universals in language for numbers and what they reveal about things like finger-counting, the use of the body, psychological processing, compounding strategies and components, and what we count. How language and numbers differ. • **Reading** – Comrie, B. (1999). Haruai numerals and their implications for the history and typology of numeral systems. In J. Gvozdanović (Ed.), *Numeral types and changes worldwide* (pp. 81–94). Berlin: Mouton de Gruyter. – Comrie, B. (2011). Typology of numeral systems. Retrieved from Max Planck Institute website: https://mpi-lingweb.shh.mpg.de/ numeral/TypNumCuhk_11ho.doc – Epps, P., Bowern, C., Hansen, C. A., Hill, J. H., & Zentz, J. (2012). On numeral complexity in hunter-gatherer languages. *Linguistic Typology*, *16*(1), 41–109. – Gordon, P. (2004). Numerical cognition without words: Evidence from Amazonia. *Science*, *306*(5695), 496–499. – Greenberg, J. H. (1978). Generalizations about numeral systems. In J. H. Greenberg (Ed.), *Universals of human language* (Vol. 3, pp. 249–295). Stanford, CA: Stanford University Press. – Overmann, K. A. (2019). *The material origin of numbers: Insights from the archaeology of the Ancient Near East*. Piscataway, NJ: Gorgias Press. (Chapters 6 & 7) • Assignment: Online comprehension quiz

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Week 7	**Cross-cultural variation 1, American number systems:** Ethnographic and linguistic analyses of numbers and subsistence strategies, climate, etc. • **Reading** – Divale, W. (1999). Climatic instability, food storage, and the development of numerical counting: A cross-cultural study. *Cross-Cultural Research*, *33*(4), 341–368. – Dixon, R. B., & Kroeber, A. L. (1907). Numeral systems of the languages of California. *American Anthropologist*, *9*(4), 663–690. – Eells, W. C. (1913a). Number systems of the North American Indians. II. Systems of numeration. *American Mathematical Monthly*, *20*(10), 293–299. – Eells, W. C. (1913b). Number systems of the North American Indians. *American Mathematical Monthly*, *20*(9), 263–272. – Greenberg, L. J. (1975). Art as a structural system: A study of Hopi pottery designs. *Studies in Visual Communication*, *2*(1), 33–50. – Moore, C. G. (1988). Mathematics-like principles inferred from the petroglyphs. *Journal of American Indian Education*, *27*(2), 30–36. • Assignment: Online comprehension quiz
Week 8	**Cross-cultural variation 2, African number systems:** How pervasive are numbers in culture? The "algebra" of kinship relations. Geometric patterns in pottery. • **Reading** – Bales, J. (2009). Fractal geometry in African American quilt traditions. In *Proceedings of the 4th Biennial Symposium of the International Quilt Museum* (pp. 1–16). University of Nebraska. – Boyd, J. P. (1969). The algebra of group kinship. *Journal of Mathematical Psychology*, *6*(1), 139–167. – Eglash, R. (1997). Bamana sand divination: Recursion in ethnomathematics. *American Anthropologist*, *99*(1), 112–122. – Ẹkundayọ, S. A. (1977). Vigesimal numeral derivational morphology: Yoruba grammatical competence epitomized. *Anthropological Linguistics*, *19*(9), 436–453. – Gilsdorf, T. E. (2012). Kinship and social relations. In *Introduction to cultural mathematics: With case studies in the Otomies and Incas* (pp. 57–72). John Wiley & Sons. – Olderogge, D. A. (1982). Counting systems in tropical and south African languages [translated from Russian]. *Trudy Instituta Etnografii*, *13*, 3–33. – Zaslavsky, C. (1999). *Africa counts: Number and pattern in African cultures*. Chicago, IL: Lawrence Hill Books. (pp. 32–51) • Assignment: Online mid-term exam.
Week 9	**Cross-cultural variation 3, Oceanian number systems:** Binary, 600 years BC (before computers), and how it emerged from counting practices. The ephemeral abacus. Body-counting and stable systems. Cultural inflections of number use. • **Reading** – Bender, A., & Beller, S. (2014). Mangarevan invention of binary steps for easier calculation. *Proceedings of the National Academy of Sciences*, *111*(4), 1322–1327. – Best, E. (1906). Māori numeration: Some account of the single, binary, and semi-vigesimal systems of numeration formerly employed by the Māori. *Transactions and Proceedings of the New Zealand Institute*, *39*, 150–180. – Núñez, R. E., Cooperrider, K., & Wassmann, J. (2012). Number concepts without number lines in an indigenous group of Papua New Guinea. *PLoS One*, *7*(4), 1–8. – Overmann, K. A. (2020). The curious idea that Māori once counted by elevens, and the insights it still holds for cross-cultural numerical research. *Journal of the Polynesian Society*, *129*(1), 59–84. – Saxe, G. B. (2012). *Cultural development of mathematical ideas*. Cambridge: Cambridge University Press. (pp. 40–48) – Wassmann, J., & Dasen, P. R. (1994). Yupno number system and counting. *Journal of Cross-Cultural Psychology*, *25*(1), 78–94. • Assignment: Online comprehension quiz
Week 10	**Cross-cultural variation 4, Social uses and motivations for numbers and their elaboration:** Social behaviors like timekeeping and counting age before and after numbers. Numbers as a

	cognitive technology. Global and local distribution patterns in numerical elaboration. • **Reading** – De Cruz, H. (2012). Are numbers special? Cognitive technologies, material culture and deliberate practice. *Current Anthropology*, *53*(2), 204–225. – Epps, P. (2006). Growing a numeral system: The historical development of numerals in an Amazonian language family. *Diachronica*, *23*(2), 259–288. – Frank, M. C., Everett, D. L., Fedorenko, E., & Gibson, E. (2008). Number as a cognitive technology: Evidence from Pirahã language and cognition. *Cognition*, *108*(3), 819–824. – Gilsdorf, T. E. (2012). *Introduction to cultural mathematics: With case studies in the Otomies and Incas*. Hoboken, NJ: John Wiley & Sons. [Chapter 1] • Assignment: Online comprehension quiz
Week 11	**Numbers in the ancient world 1, Prehistoric, proto-European, and contemporary European numbers:** The idea of numbers as "entities" or true objects in themselves and its emergence in the Renaissance. • **Reading** – d'Errico, F. (1991). Microscopic and statistical criteria for the identification of prehistoric systems of notation. *Rock Art Research*, *8*, 83–93. – Jègues-Wolkiewiez, C. (2005). Aux racines de l'astronomie, ou l'ordre caché d'une oeuvre paléolithique. *Antiquités Nationales*, *37*, 43–62. [English translation] – Marshack, A. (1991). The Taï plaque and calendrical notation in the Upper Palaeolithic. *Cambridge Archaeological Journal*, *1*(1), 25–61. – Overmann, K. A. (2014). Finger-counting in the Upper Palaeolithic. *Rock Art Research*, *31*(1), 63–80. – Schlimm, D., & Neth, H. (2008). Modeling ancient and modern arithmetic practices: Addition and multiplication with Arabic and Roman numerals. In *Proceedings of the 30th annual conference of the Cognitive Science Society* (pp. 2097–2102). Austin, TX: Cognitive Science Society. • Assignment: Online comprehension quiz
Week 12	**Numbers in the ancient world 2, Mesopotamia and Babylon:** Neolithic clay tokens and sexagesimal numbers. One- and two-dimensional representations of number. Wholes and parts: fractions, fractional representation, and subitizing. How many types of addition and subtraction are there? The role of writing in numerical elaboration. • **Reading** – Chrisomalis, S. (2005). Evaluating ancient numeracy: Social versus developmental perspectives on ancient Mesopotamian numeration. *Annual Meeting of the Jean Piaget Society (Vancouver, British Columbia)*, 1–21. – Lewy, H. (1949). Origin and development of the sexagesimal system of numeration. *Journal of the American Oriental Society*, *69*(1), 1–11. – Overmann, K. A. (2019). *The material origin of numbers: Insights from the archaeology of the Ancient Near East*. Piscataway, NJ: Gorgias Press. (Chapter 10) – Schmandt-Besserat, D. (2001). Feasting in the Ancient Near East. In M. Dietler & B. Hayden (Eds.), *Feasts: Archaeological and ethnographic perspectives on food, politics, and power* (pp. 391–403). Tuscaloosa, AL: University of Alabama Press. – Schmandt-Besserat, D. (2010). The token system of the Ancient Near East: Its role in counting, writing, the economy and cognition. In C. Renfrew & I. Morley (Eds.), *The archaeology of measurement: Comprehending heaven, earth and time in ancient societies* (pp. 27–34). Cambridge: Cambridge University Press. • Assignment: Calculating with Neolithic clay tokens and Babylonian sexagesimal numbers.
Week 13	**Numbers in the ancient world 3, Egypt, Greece, and Rome:** Egyptian and Greek numbers. The "discrete" and the "continuous" or the difference between counting "things" and "lengths." Roman numerals and abacus. The century-long argument between abacists and algorists. • **Reading** – Damerow, P. (2010). *Abstraction and representation: Essays on the cultural evolution of thinking*. Dordrecht, The Netherlands: Kluwer Academic. (pp. 46–60)

	− Klein, J. (1992). *Greek mathematical thought and the origin of algebra*. New York: Dover Publications, Inc. − Schlimm, D., & Neth, H. (2008). Modeling ancient and modern arithmetic practices: Addition and multiplication with Arabic and Roman numerals. In B. C. Love, K. McRae, & V. M. Sloutsky (Eds.), *Proceedings of the Cognitive Science Society* (Vol. 30, pp. 2097–2102). Austin, TX: Cognitive Science Society. − Stone, W. E. (1972). Abacists versus algorists. *Journal of Accounting Research, 10*(2), 345–350. • Assignment: Online comprehension quiz
Week 14	**Numbers in the ancient world 4, China:** Chinese numbers and the abacus. Cultural ideas about numbers and luck. Chinese lexical and notational numbers, "transparency," and mathematical task performance. • **Reading** − Cantlon, J. F., & Brannon, E. M. (2007). Adding up the effects of cultural experience on the brain. *Trends in Cognitive Sciences, 11*(1), 1–4. − Pellatt, V. (2007). *Numbers and numeracy in Chinese culture, language, and education: The social substratum of the development of mathematical thinking*. Lewiston, NY: Edwin Mellen Press. − Siegler, R. S., & Mu, Y. (2008). Chinese children excel on novel mathematics problems even before elementary school. *Psychological Science, 19*(8), 759–763. • Assignment: Calculating with an abacus.
Week 15	**Numbers in the ancient world 5, Mesoamerica and South America:** Maya numbers, Maya abacus (Nepohualtzintzin), and calendrical time. The Inca khipu and yupana. How numbers are represented and organized cross-culturally in written notational form. Whither numbers? • **Reading** − Andina. (2014, June). Archaeologists find 25 quipus at Inca site in Peru. *Living in Peru*. Retrieved from https://www.livinginperu.com/news-archaeologists-find-25-quipus-at-inca-site-in-peru-103338/ − Ascher, M., & Ascher, R. (1981). *Code of the quipu: A study in media, mathematics, and culture*. Ann Arbor, MI: University of Michigan Press. − Comrie, B. (2005). Endangered numeral systems. In J. Wohlgemuth & T. Dirksmeyer (Eds.), *Bedrohte Vielfalt: Aspekte des Sprach(en)tods* (pp. 203–230). Berlin: Weißensee-Verlag. − Kaplan, R. (2000). *The nothing that is: A natural history of zero*. New York: Oxford University Press. (Chapter 8) − Pica, P., Lemer, C., Izard, V., & Dehaene, S. (2004). Exact and approximate arithmetic in an Amazonian indigene group. *Science, 306*(5695), 499–503. • Assignment: Representing numbers with a khipu • Assignment: Online final exam

POLICIES (READ THE FINE PRINT!)

Attendance and participation

Students are expected to complete all lessons, assignments, and exams. Students who choose to withdraw from the course are responsible for completing the necessary withdrawal forms. Students who stop attending without formally withdrawing still appear on the final grade roster and must be assigned a grade. Consult the schedule for additional information and withdrawal dates.

Preparation

Students should prepare for the weekly lesson by reading the material listed for each session. Students should have a general grasp of the relevant ideas introduced in the material. Students should be prepared to apply reading assignments and material presented in previous weeks to assignments and exams.

Academic Conduct

Students should read the University Bulletin and make note of campus policies regarding breaches of the honor system and academic honesty. Misuse of academic materials will not be tolerated. For additional information on academic conduct, see the university website: http://www.uccs.edu/Documents/dos/Final%20Approved%20Code%20of%20Conduct%20Fall%202016%20.pdf.

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Deadlines, Due Dates, Etc.

Students should notify the instructor immediately of any extenuating circumstances which do not allow them to meet course deadlines, due dates, or exams.

Reading Assignments

Reading assignments comprise the text and majority of instructional material. The complete bibliography (with optional readings) will be available upon request.

Exams

Tests for the course will be based on the material presented in the weekly assignments. Exams will consist of online multiple-choice, short answer/matching, and essay questions. Exams will be graded on a 100-point scale: A (90–100), B (80–89), C (70–79), D (60–69), F (59 and below), with option for plus (upper portion of range) and minus (lower portion of range).

Writing quality

Written responses to exam questions will be graded on thoughtfulness, including attention to the detail of the assigned question and the ability to cite examples from reading assignments. Points awarded for essay responses are as follows: 30% Clarity (clear, concise, understandable writing); 30% Organization (thesis statement, topic sentences, development following the thesis, a conclusion that does not introduce new ideas), and 40% Support (uses examples from the course material that are relevant to the original question).

Graduate students

Graduate students will complete a 10–12-page research paper on a topic approved by the instructor, and in addition will have an extra essay response on each exam.

Disabilities

Students requesting an accommodation should contact Disability Services. Please advise the instructor of any special needs. Please submit any required forms prior to the exams. For additional information on disability services, see the university website: http://www.uccs.edu/ disability/facultystaff/faculty-accommodation-letter.html.

Military Service

Students who are active or reserve military with the potential of being called to service and/or training during the course should coordinate with the instructor during the first week to discuss accommodations. For additional information on military student support, see the university website: http://www.uccs.edu/military/current-students/active-duty-and-reservists.html.

Communication

Dr. Overmann's primary method of communicating with students is by email. She checks her email compulsively and expects students to check theirs regularly. Consistent with UCCS policy, university email addresses will be used (e.g., student@uccs.edu).

Useful University Phone Numbers (719) 255-

Academic Advising 3260; Anthropology Department 3620; Disability Services 3354; LGBTQ Resource Center 3447; Library 3296; Public Safety 3111; University Testing Center 3354; Veteran and Military Student Affairs Office 3253; Veterans Health and Trauma Clinic 8003; Wellness Center Mental Health Services 4444; Writing Center 4336.